

SEMINAR SOMMERSEMESTER 2017

ALGEBRAIC GEOMETRY OVER C^∞ -RINGS

We all know the history of calculus: the imprecise and heuristic reasonings involving *infinitesimals*, the long struggle to put it on a firm footing, ending with the rigorous definition of the limit due to Weierstraß. Analysis became then a powerful tool to study objects which look locally like \mathbb{R}^n : manifolds and orbifolds.

However, another approach is possible: to make the infinitesimals rigorous. This is the approach taken by the *synthetic differential geometry* which replaces the real line \mathbb{R} by R - the real line + infinitesimals. R is not a field, but rather a (local) ring with nilpotents. Synthetic geometers are interested in proving theorems about manifolds using the methods based on formalised infinitesimals. However the category of spaces which can be built based on the ring R is much wider: it includes the C^∞ -schemes and C^∞ -stacks - objects generalising manifolds and orbifolds, respectively. This kind of objects appear very naturally, even if one is interested only in smooth manifolds, since, for example, spaces parametrising manifolds satisfying certain conditions will often no longer be manifolds, but such generalised objects.

Recently C^∞ -schemes appeared in the context of *derived differential geometry* - the differential analogue of the derived algebraic geometry. The idea is that objects which appear very singular in the ordinary differential geometry are really truncations of objects which are in some sense smooth, but live in a higher category. Derived geometry is one of the fastest growing branches of mathematics, with numerous applications in algebraic geometry, topology and symplectic geometry.

The aim of the seminar is to read and understand together the article of D. Joyce *Algebraic Geometry over C^∞ -rings* in which the foundations of the theory of C^∞ -schemes and of C^∞ -stacks are developed. This will provide a solid basis for further study of derived geometry.

Literature:

Dominic Joyce, *Algebraic Geometry over C^∞ -rings*,
available at <https://arxiv.org/abs/1001.0023>

Intended audience/participants: Students MSc Mathematics or Physics.

Prerequisites: Basic algebra (knowledge of basic concepts in category theory *is* necessary, but can be quickly acquired by reading §11 of Chapter I of Lang's *Algebra*).

Place and Time: Thursdays, 10-12, room 016(1310) in the Mensa

First meeting: Thursday, April 13, 10-12, room 016(1310) in the Mensa