Analysis and PDE 2024
Book of Abstracts

Analysis and PDE 2024

September 30 – October 2, 2024
Leibniz Universität Hannover

Invited speakers:

Fabrice Baudoin (Aarhus)
Miroslav Bulíček (Prague)
Li Chen (Mannheim)
Clotilde Fermanian Kammerer (Paris)
Colin Guillarmou (Paris)
Carolin Kreisbeck (Eichstätt-Ingolstadt)
Marko Lindner (Hamburg)
Paolo Marcellini (Florence)
Irina Markina (Bergen)
El Maati Ouhabaz (Bordeaux)
Luca Rizzi (Trieste)
Roland Schnaubelt (Karlsruhe)
Stefanie Sonner (Nijmegen)
Emmanuel Trélat (Paris)
Christiane Tretter (Bern)
Tobias Weth (Frankfurt)
Petra Wittbold (Duisburg-Essen)

Organizing Committee:

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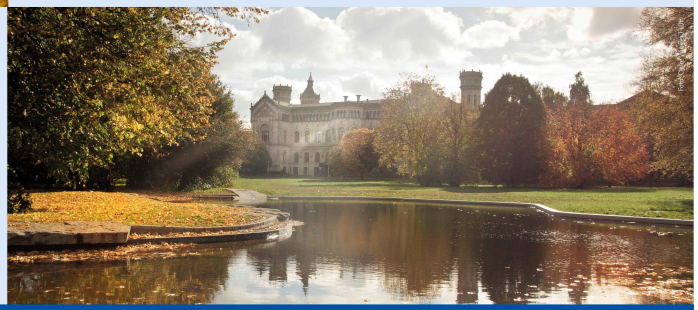

www.math-conf.uni-hannover.de/anapde24.html

Contents

Fabrice Baudoin

Bottom of the spectrum for manifolds foliated by minimal leaves

Aarhus University

We give a sharp estimate for the bottom of the spectrum of a Riemannian manifold which is foliated by minimal leaves and transversally negatively curved. Our proof, which uses probabilistic methods, also yields an estimate for the bottom of the sub-Riemannian spectrum.

Miroslav Bulíček

On the well-posedness of time-dependent three-dimensional activated Euler fluid flow

Charles University

In this talk we present the mathematical properties of time-dependent flows of incompressible fluids that respond as an Euler fluid until the modulus of the symmetric part of the velocity gradient exceeds a certain, a priori given but arbitrarily large, critical value. Once the velocity gradient exceeds this threshold, a dissipation mechanism is activated. Assuming that the fluid, after such an activation, dissipates the energy in a specific manner, we prove that the corresponding initial-boundary-value problem is globally-in-time well-posed in the sense of Hadamard. We further discuss the potential of using the activated fluids.

Li Chen

Mean field control for diffusion aggregation system with Coulomb interaction

Universität Mannheim

In this talk, I will talk about the mean field control problem for a multi-dimensional diffusion-aggregation system with Coulomb interaction. The existence of optimal control is proved through the Gamma convergence of the control problem of a regularized particle control problem. The optimal control problem on the particle level is studied by using the corresponding Liouville equation. Because of strong aggregation effect, additional difficulties arises from control function in the well-posedness theory, so that the known method for multi-dimensional Keller-Segel equation can not be directly applied. We use a combination of local existence result and boot-strap argument to obtain the global solution with small initial data. Since we obtain a strong propogagtion of chaos result by combining the convergence in probability and relative entropy method, the compact support requirement for control functions, which has been often used in the literature, is not need. The talk is based on a joint work with Yucheng Wang and Zhao Wang.

Eske Ewert

Shubin calculus on graded Lie groups

Leibniz University Hannover

We introduce a new Shubin type pseudodifferential calculus for graded Lie groups such as the Heisenberg group. Characteristic for calculi on these groups is that the order of left-invariant differential operators is determined by the grading of the Lie algebra. The role of elliptic operators is then played by Rockland operators. Using this adapted order for left-invariant vector fields together with the homogeneous order of polynomials, one obtains a Shubin type calculus for differential operator with polynomial coefficients.

We construct a surrounding pseudodifferential calculus and establish conditions for global hypoellipticity. It can be employed, for instance, to study Rockland operators with potentials. Moreover, we show mapping properties of the operators between corresponding Sobolev spaces and investigate when they are Fredholm. These results provide a foundation for further investigations into the spectral theory and index theory of operators on graded Lie groups. The talk is based on joint work with Philipp Schmitt.

Clotilde Fermanian Kammerer

Semi-classical analysis on graded groups and applications

Université d'Angers

In this talk we will explain how to develop a semi-classical setting on graded Lie groups in terms of pseudodifferential operators, semiclassical measures, Wick quantization and coherent states. We will present applications to propagation of singularities in simple cases.

Mario Fuest

Finite-time blow-up in fully parabolic quasilinear Keller–Segel systems with supercritical exponents

Leibniz University Hannover

The fully parabolic quasilinear Keller–Segel system

$$\begin{cases} u_t = \nabla \cdot ((u+1)^{m-1}\nabla u - u(u+1)^{q-1}\nabla v), \\ v_t = \Delta v - v + u, \end{cases}$$

which we consider in a ball $\Omega \subset \mathbb{R}^n$, $n \geq 2$, admits unbounded solutions whenever $m, q \in \mathbb{R}$ satisfy $m - q < \frac{n-2}{n}$. These are necessarily global in time if $q \leq 0$ and finite-time blow-up is known to be possible if $q > 0$ and $\max\{m, q\} \geq 1$. Utilizing certain pointwise upper estimates for u, we are able to give an affirmative answer to the (for nearly a decade formerly open) question whether solutions may blow up in finite time if $\max\{m, q\} < 1$. If $n = 2$, for instance, we construct solutions blowing up in finite time whenever ($m - q < 0$ and) $q < 2m$.

Colin Guillarmou

Marked length spectrum rigidity for Anosov surfaces

Université Paris-Saclay

We give a proof of the fact that the marked length spectrum of a surface (dim 2) with Anosov geodesic flow determines the isometry class of the metric. Joint work with Lefeuvre and Paternain.

Carolin Kreisbeck

Variational analysis of integral functionals involving nonlocal gradients

Katholische Universität Eichstätt-Ingolstadt

Motivated by new nonlocal models in hyperelasticity, we discuss a class of variational problems with integral functionals depending on nonlocal gradients, precisely, on truncated versions of the Riesz fractional gradient. We address several aspects regarding the existence theory of these problems and their asymptotic behavior. Our analysis relies on suitable translation operators that allow us to switch between nonlocal and classical gradients and are thus helpful technical tools for transferring results from one setting to the other. Based on this approach, we show that quasiconvexity, the natural convexity notion in the classical calculus of variations, characterizes weak lower semicontinuity also in the nonlocal setting. As a consequence of a general Gamma-convergence statement, we derive relaxation and homogenization results. Analyzing the limiting behavior as the fractional order tends to 1 or the horizon tends to 0 yields localization to a classical model. This is joint work with Javier Cueto (Universidad Autónoma de Madrid) and Hidde Schönberger (KUEI).

Marko Lindner

Localisation of spectra and pseudospectra

Technische Universität Hamburg

Given a bounded and non-normal finite propagation operator A on $\ell^2(\mathbb{Z})$, an integer $n \in \mathbb{N}$ and some $\varepsilon > 0$, we show that every ε-pseudoeigenvector of A has a subinterval of length n that is an $(\varepsilon + \varepsilon_n)$-pseudoeigenvector of the corresponding restriction of A, where $\varepsilon_n \sim 1/n$ – and we know the proportionality constant quite well.

As a consequence, we cover the pseudospectrum of A by a union of pseudospectra of its restrictions to intervals of length n, proving the absence of (pseudo)spectrum outside that union. We extend the result

- to spectra,
- to essential spectra,
- to the half-axis and the interval,
- to discrete Abelian groups G, and
- we derive the optimal ε_n as a Dirichlet eigenvalue of an associated graph Laplacian.

Paolo Marcellini

The Leray-Lions existence theorem under general growth conditions

Università di Firenze

We discuss about the celebrated existence theorem published in 1965 by *Jean Leray* and *Jacques-Louis Lions*. As well known, it is an existence result of weak solutions to a class of Dirichlet problems for second order nonlinear elliptic equations under the so-called natural growth conditions.

We describe the possibility to adopt assumptions which allow to handle a class of *nonuniformly elliptic PDEs* satisfying general growth conditions, such as for instance $p, q-$*growth*, nowadays largely studied in the literature. In particular we describe some existence and regularity results recently obtained in collaboration with *G.Cupini and E.Mascolo*, to appear in the *Journal of Differential Equations*.

Irina Markina

Stokes graphs of a quadratic differential related to a Rabi model

Universitetet i Bergen

To study the behaviour of solutions to a second-order linear differential equation $y''(t) + Q(z,t)y = 0$ one can associate the quadratic differential $Q(z)dzdz$ on the punctured Riemann sphere and consider its Stokes graph. We consider an ODE related to a Rabi problem describing a light-atom interaction in physics. The associated quadratic differential is meromorphic with two finite poles. The integrability condition for this type of ODE under isomonodromic deformations is related to a non-linear second-order differential equation, known as Painlevé V. In my talk, I will explain a classification of the Stokes graphs according to the nature of the zeros of the meromorphic quadratic differential originated in the Rabi model. This is a joint work with R. Langøen (University of Bergen) and A. Solynin (Texas Tech, USA).

El Maati Ouhabaz

An almost Riesz transform inequality for general Riemannian manifolds

Université de Bordeaux

Given a general complete Riemannian manifolds M and denote by ∇ and Δ the gradient and the Laplace-Beltrami operator. We study boundedness on $L^p(M)$ of the Littlewood-Paley-Stein functionals

$$\left(\int_0^\infty |\nabla F(t\Delta)f|^2\, dt\right)^{1/2}$$

for holomorphic functions F on some sector and which have a decay at infinity. From this we deduce the following multiplicative inequality

$$\|\nabla f\|_p^2 \leq C_\epsilon \|\Delta^{\frac{1}{2}+\epsilon} f\|_p \|\Delta^{\frac{1}{2}-\epsilon} f\|_p$$

for all $p \in (1, 2]$.

Luca Rizzi

Sard properties for polynomial maps in infinite dimension with applications to the Sard conjecture in Carnot groups

SISSA Trieste

Sard's theorem asserts that the set of critical values of a smooth map from one Euclidean space to another one has measure zero. It is well-known, however, that when the domain is infinite dimensional and the range is finite dimensional the property may fail. I will present recent results with A. Lerario and D. Tiberio (SISSA), establishing sharp quantitative criteria for the validity of Sard's theorem in this setting. Our motivation comes from sub-Riemannian geometry and we provide in particular applications of our results to the Sard conjecture on Carnot groups.

Lina Schmitz

A soap film bridge driven by an electrostatic force

Leibniz University Hannover

I discuss a free boundary problem describing a tubular soap film bridge subjected to an electrostatic force. For the rotationally symmetric case, I show local well-posedness of this problem by recasting it as a single quasilinear parabolic equation with a non-local source term. Subsequently, I focus on qualitative properties of stationary solutions and prove, in particular, that the soap film deflects monotonically outwards if the strength of the electrostatic force is increased.

Roland Schnaubelt

Strichartz estimates for Maxwell equations

Karlsruhe Institute of Technology

Wave-type equations typically exhibit dispersive behavior which means that wave packets smear out if time evolves. These properties are quantified by Strichartz estimates for solutions to linear problems. Compared to the preservation of L^2-based norms, they provide increased spatial integrability of solutions at the price of decreased time integrability and (for the wave or Maxwell equations) of a loss of regularity. Since the 90's these and related estimates have been crucial for the tremendous progress in the wellposedness and qualitative theory of semilinear dispersive problems.

We first give an overview on Strichartz estimates for the wave equation on $\mathbb{R}^d$. Some applications to the wellposedness theory for semilinear wave equations will be indicated. We then focus on recent results obtained jointly with Robert Schippa (Berkeley) and Piero D'Ancona (Rome) for the Maxwell system on $\mathbb{R}^3$ or $\mathbb{R}^2$, where the system character provides several new challenges. Applications to quasilinear and certain semilinear systems will be sketched.

Stefanie Sonner

Degenerate reaction diffusion describing biofilm growth

Radboud University Nijmegen

Biofilms are dense aggregations of bacterial cells attached to a surface and held together by a self-produced slimy matrix. We consider models for spatially heterogeneous biofilms that are formulated as quasilinear reaction diffusion systems. Their characteristic feature is the two-fold degenerate diffusion coefficient for the biomass density comprising a polynomial degeneracy (as the porous medium equation) and a fast diffusion singularity as the solution approaches its maximum value. This degenerate equation is coupled to a semilinear parabolic equation or an ordinary differential equation for the nutrient concentration. We discuss results on the well-posedness and regularity of solutions. For couplings to ordinary differenial equations we also prove the existence of traveling wave solutions.

Emmanuel Trélat

Spectral analysis of sub-Riemannian Laplacians and Weyl measure

Sorbonne Université

In collaboration with Yves Colin de Verdière and Luc Hillairet, we study spectral properties of sub-Riemannian Laplacians, which are selfadjoint hypoelliptic operators satisfying the Hörmander condition. Thanks to the knowledge of the small-time asymptotics of heat kernels in a neighborhood of the diagonal, we establish the local and microlocal Weyl law. When the Lie bracket configuration is regular enough (equiregular case), the Weyl law resembles that of the Riemannian case. But in the singular case (e.g., Baouendi-Grushin, Martinet) the Wey law reveals much more complexity. In turn, we derive quantum ergodicity properties in some sub-Riemannian cases.

Christiane Tretter

Challenges for non-selfadjoint spectral problems in analysis and computation

Universität Bern

Non-selfadjoint spectral problems appear frequently in a wide range of applications. Reliable information about their spectra is therefore crucial, yet extremely difficult to obtain. This talk focuses on tools to master these challenges such as spectral pollution or spectral invisibility. In particular, the concept of essential numerical range for unbounded linear operators is introduced and studied, including possible equivalent characterizations and perturbation results. Compared to the bounded case, new interesting phenomena arise which are illustrated by some striking examples. A key feature of the essential numerical range is that it captures, in a unified and minimal way, spectral pollution which may affect e.g. spectral approximations of PDEs by projection methods or domain truncation methods. As an application, Maxwell's equations with conductivity will be considered.

(Joint work with S. Boegli, M. Marletta, and also F. Ferraresso)

Tobias Weth

The Schiffer problem on the cylinder and on the 2-sphere

Goethe Universität Frankfurt

I will discuss a new result on the existence of a family of compact subdomains of the flat cylinder for which the Neumann eigenvalue problem for the Laplacian admits eigenfunctions with constant Dirichlet values on the boundary. These domains have the property that their boundaries have non-constant principal curvatures. In the context of ambient Riemannian manifolds, our construction provides the first examples of such domains whose boundaries are neither homogeneous nor isoparametric hypersurfaces. The underlying functional analytic approach we have developed overcomes an inherent loss of regularity of the problem in standard function spaces. With the help of this approach, we also construct a related family of subdomains of the 2-sphere. By this we disprove a conjecture of Souam from 2005. This is joint work with M.M. Fall and I.A. Minlend.

Petra Wittbold

On weak and entropy solutions of generalized time-fractional porous medium type equations

Universität Duisburg Essen

We present results on existence and uniqueness of weak and entropy solutions to a degenerate quasilinear subdiffusion problem of porous medium type with bounded measurable diffusion coefficients that may explicitly depend on time. The integro-differential operator in the equation includes, in particular, the time-fractional derivative case. A key ingredient in the proof of existence is a new compactness criterion of Aubin-Lions type which involves the non-local in time operator.